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Temperature Fluctuations for a Platoon of Vehicles in Contact with a Heat Bath

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Abstract

Traffic flow is often thought of highly stochastic multi degrees of freedom event. Studies have shown that it has mesoscopic dynamics, and in fact rarely a Boltzmann-Gibssian type statistical mechanics. Instead, traffic flow should be handled within a nonadditive and possibly nonextensive statistical mechanics framework. A platoon formation is a well-organized collection of vehicles. The idea behind the platoon of vehicles is that the efficient use of the infrastructure may greatly be enhanced if there is an increase in the cooperation and organization. A well-controlled platoon of vehicles is a highly organized body of particles where the speed of the vehicles or intervehicle spacing is aimed to be held constant during the regular cruising conditions. It is claimed that a well-organized and controlled platoon of vehicles is in fact a system which is weakly coupled to a heat bath, forming the canonical ensemble. Authors propose that a platoon of vehicles at constant mean speed, even with fluctuations, be equivalent to a system with finite degrees of freedom (dof) weakly coupled to a much larger dof reservoir whose temperature variance is a function of its dof. An equivalent number of vehicles concept is devised as a function of the reciprocal of the temperature variance. It is shown that as long as the reservoir dof is much larger than the platoon dof, the finite reservoir may then safely be assumed as an infinite reservoir.

1. Introduction

Temperature fluctuations have been an ardent discussion topic for several decades. The old school of thermostatistics has constructed the idea of canonical ensemble so that system is fixed as thermometer when weakly coupled to a thermostat. This general view has simplified the notion of temperature of systems at least under infinite size reservoirs [1, 2]. But another school of the same discipline rightfully argues that one has a thermometer whose temperature is fluctuating [3, 4]. The scope of the general view on temperature fluctuations is elaborated in sections 1 and 2.

The idea that the system and the heat bath could and should exchange heat would unfortunately bring about not just temperature but energy fluctuations on finite systems as well. While the energy and the temperature are considered coupled for small systems, they are not so for infinite reservoirs.

Studies have shown that vehicular traffic has mesoscopic dynamics [5] and in fact, rarely, a Boltzmann-Gibbsian (BG) type statistical mechanics [6-8]. When the multi-component systems have interacting particles or subsystems, it is known that the system dynamics deviates from BG thermostatistics. This paper handles the quite interesting problem of the temperature fluctuations of the vehicle platoons in the context of (nonadditive) statistical mechanics. We claim that any platoon formation coupled to the finite or infinite size heat bath, experiences the temperature (speed) fluctuations. The fluctuations are minimum when the heat bath is an infinite one. In this paper, we also claim that depending on the quality of tracking control of vehicles in a given platoon, a so-called virtual number of vehicles must be specified for platoons of vehicles, inversely proportional to the temperature fluctuations.

As for the link between temperature and the velocity, there are various approaches. One of these approaches [9], specifies a well-defined velocity variance as some generalized temperature. This reasoning makes sense when the Hamiltonian is defined over the mean velocity. But when one speaks of an ideal platoon formation, the mean velocity is shared by all, so that when a certain kinetic energy is present, variance, hence temperature fluctuations would have been zero, where temperature is obviously not zero. Another way of thinking may be found in [10], which is also quite appropriate for a system of vehicles where the temperature is taken to be the mean speed (e.g. the specified

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speed of the platoon). In this paper, the authors follow this latter way of thinking. It will be justified later in that a stabilized platoon of vehicles may serve as a heat bath, as well as a system, which has a fixed, and fluctuating temperature, respectively.

It could be noticed that the arguments on the connection between the temperature fluctuations and nonextensivity framework have intensified recently e.g. [11-13]. In this regard, setting the formulations of the temperature fluctuations of platoons of vehicles is missing in the literature. In this paper, the temperature fluctuations are handled for a platoon of vehicles in contact with a heat bath. The vehicular traffic literature indicates that there are quite a few studies on intra-platoon vehicles. The general tendency of the studies is on e.g. the control and communication of the platoon vehicles, efficient use of capacity and decreasing fuel consumption on highways. In the literature, recently for example, the authors [14] propose a mechanism which autonomously selects a platoon leader, eliminating the dependence on the automated highway system. The velocity and position information of the neighboring vehicles are considered in this process. Their aim is to increase capacity, and decrease the fuel consumption and travel time of the vehicles. Their mechanism is employed in a simulation environment and its performance is discussed. Another study [15] is concerned with the speed planning in regard to real-time fleet management. The efficiency of the proposed fast algorithm is discussed and the linkage between the energy savings and speed planning is explained. In the study [16], a consensus based control approach for vehicle platooning is proposed. Different from a static control topology, the authors deal with a flexible control system. Heterogeneous and time-varying communications delays are examined. Stability and convergence analyses are given, and the robustness of the approach is interpreted in the study. Such examples of the vehicular platoon studies could be expanded. However, so far, temperature fluctuations have not been handled in the vehicular platoon formations coupled to a heat bath in the literature.

The authors in this paper examine the methods governing the temperature fluctuations literature, and propose the methodological prescription for the temperature fluctuations in the vehicular platoon formations. Platoon-based systems in contact with finite or infinite reservoirs are considered, and their set of principles is described and discussed.

2. Schools of Thought on Temperature Fluctuations in Canonical Ensembles

The problem of the temperature fluctuations has been a thorny issue since the canonical ensemble framework had been formulated by Gibbs, following the original work by Boltzmann, [17]. It may safely be said that the original idea with the canonical ensemble is to have a small system, weakly coupled to a heat bath, as a thermometer, where the temperature of the latter is fixed. For the elucidation of the temperature fluctuations in canonical ensembles, this section exhibits a certain number of schools of thought as follows.

In literature, a considerable number of studies deals with the temperature fluctuations and a clash of ideas has emerged. Of those studies, e.g. a widely known study [18] interprets the temperature fluctuations in detail and, for example, exhibits a common equation which involves mean square fluctuations of temperature and heat capacity. In another representative study [19], the authors measured the temperature fluctuations in canonical ensemble with experiments.

For the temperature fluctuations, one more notable example is the work done by Kittel who emphatically asserted the notion of energy fluctuations in the systems in canonical ensembles, excluding those of temperature [1, 2]. Kittel [2] states that temperature is unambiguously defined for a system in thermal equilibrium with a heat bath and argued that the temperature fluctuations would not occur and temperature is fixed in the canonical ensemble, while energy fluctuations would do. Kittel [2] also states that it is inappropriate to fiddle with the orthodox view of the Gibbs and others. However, there had been a collection of objections since then, two of which belong to Mandelbrot [20] where the temperature of the system would stem from the estimates, by designating temperature fluctuation as “a well-defined and unavoidable notion”, and McFee [3].

The approach as advocated by both Mandelbrot [20] and Falcioni et al. [21] considers temperature somewhat ill-defined. For small systems, in Mandelbrot’s own words [20], a fog of uncertainty is unavoidable. One good method then is to measure the system energy in order to estimate the heat bath temperature, which translates that the problem of determination of the temperature of a small system is nothing but an estimation problem that could be handled within the estimation theory [22]. Falcioni et al. [21] also addressed that the temperature fluctuations would be based on the temperature estimates. The authors express that once the number of particles grows to infinity, the uncertainty in the estimate of temperature would tend to zero and the system could also be thought as a reservoir with a definite temperature. Furthermore, when Lindhard [23] discusses “the assertion of Rosenfeld” with respect to the relationship between the fluctuations of temperature and energy, the author mentions that the canonical ensemble would possess an exact temperature.

Mandelbrot [20], as mentioned in [21], has pointed out that estimation theory may be of help at specifying the temperature of a thermostat, with a sufficient number of measurements.

$$\frac{\Delta\hat{T}}{T} = \frac{\sqrt{\varphi_T^2}}{T} = \frac{1}{\sqrt{\aleph(N+1)}} \tag{1}$$

where $\aleph$ is the independent measurements, N stands for the number of particles and $\Delta\hat{T}$ defines the fluctuation estimate [21].

Another school takes the temperature fluctuations as a natural extension of statistical mechanics when system size becomes an issue. Spontaneous energy exchange between the system and the thermostat is what makes the temperature of the small system fluctuate, whereas for the heat bath this exchange is negligible, such as [3] and [24]. McFee [3] studies temperature fluctuations on small systems and argues that there is a relationship between the system size and temperature i.e. as the system grows, its average temperature tends to fixed temperature. McFee [4] also discusses average temperature and comments that temperature will fluctuate, but average of the inverse temperature will correspond to inverse of the reservoir temperature, and thus it would be invariant. Stodolsky [25] also discussed the temperature fluctuations of a system in connection with its total heat capacity considering multiparticle production.

Recently, the authors [22] listed the different views on temperature fluctuations. The authors address the notion of temperature fluctuations within especially the ideas of Landau & Lifshitz [18] and Kittel [1, 2]. Moreover, the other standpoint in [22] is also related with the temperature estimation that can be understood as a fluctuation [20, 21]. In the literature, temperature fluctuations may be formulated as in [18, 19, 22] in line with [24], such that

$$\frac{<(\Delta T)^2>}{\langle T_r\rangle^2}=\frac{k}{NC_v^0} \tag{2}$$

where T_r is the reservoir temperature, $<\cdots>$, the canonical ensemble average, k, Boltzmann's constant, C_v^0, the heat capacity (per particle) under constant volume and ΔT denotes the fluctuation of the reservoir temperature from the canonical temperature.

In another typical study [26], it is stated that the problem of temperature fluctuations of small systems in canonical systems may be traced back to Brownian motion. In their study, the authors also argue the equilibrium thermodynamics of a mesoscopic system in contact with a heat bath at a given temperature. Specific system configurations and time are considered in the evaluation of energy and temperature fluctuations, and the ergodic composition is one of the main arguments in that paper.

It is commonly expected that in equilibrium statistical mechanics the thermostat temperature would be invariant when a subsystem coupled with the infinite heat bath. However, when taking into account finite heat bath, Prosper [24] expressed that the temperature fluctuations would be pronounced. In this respect, systems could be in contact with finite heat baths and their thermostatistics would implicate power law distributions such as in [11]. The connection between the temperature fluctuations and nonextensivity parameter q is exhibited e.g. [13, 27, 28].

The number of particles i.e. dofs significantly influences the fluctuations. Potiguar & Costa [12] consider that systems coupled with finite heat baths have power law distributions. With respect to this argument, for example, Adib et al. [29] state that the violation of the condition of infinite heat bath capacity follows the Tsallis thermostatistics. The derivative of the heat bath inverse temperature with respect to energy is related with the Tsallis q parameter, which implies the power law canonical distributions [29-31]. Once the infinite heat capacity conditions are in question, the q parameter would be equal to unity, covering the BG canonical distribution. The most recent paper [8] asserts that the q parameter of the perfectly coordinated platoon of the moving vehicles matches the unity, which would implicitly propound that the platoon is being transformed from a small system to an infinite thermostat.

The recent work [13] deals with the general deformed entropy formula considering the finite reservoir with fluctuations. The huge fluctuations are evaluated in the limit, and the canonical probability distribution function is at issue and for high probabilities the distribution matches the Boltzmannian exponential function, whereas for low probabilities it approaches the cumulative Gompertz one. The macroscopic thermodynamics of the system in contact with the heat bath in equilibrium is presented from the perspective of power law distributions [32]. Wilk & Włodarczyk [33] point out that the connection between the nonextensivity parameter q and some specific intrinsic fluctuations of the temperature in the hadronizing system. Power-like behavior is discussed within the context of fluctuations in a multiparticle production process. The equivalence of temperature and volume fluctuations is indicated when the total energy is held constant. One may also find the relationship between the parameter q and relative variance of inverse temperature for example in [34].

3. Setting the Stage for the Vehicular Platoon Temperatures

3.1. Methodological Background

A platoon of vehicles in traffic, weakly coupled to an imaginary traffic reservoir could be formed as a canonical ensemble where the platoon experiences temperature (speed) fluctuations. As the speed fluctuations tend to zero, a system of the platoon of vehicles begins to act like a reservoir itself, becoming an infinite degree of freedom reservoir at zero fluctuations. In this paper, the authors set the vehicular platoon temperatures, formulating the effective number particles at first.

The formulation of the reservoir and system (platoon) temperatures are defined, and the relationship between them is proposed in terms of Tsallis entropic indices. The mathematical difference of the average temperatures of the platoon and finite reservoir is also expressed.

As displayed in the following section, the platoon formation is characterized. The speed of the vehicular platoon formation is identified as temperature, and the speed and number of vehicles become the focal variables for the preparation of methodological prescription of this paper.

Interaction and information exchange in the framework of nonextensive thermostatistical reasoning are also adapted in the temperature fluctuations. This is also related with the violation of the infinite heat capacity. In this context, if a perfectly controlled platoon formation emerges, the platoon itself starts acting like infinite heat bath whose temperature does not fluctuate. This is also a limiting case for the current approach.

Furthermore, once a finite reservoir is sufficiently larger than a platoon coupled to it, then the larger reservoir acts like an infinite one. The formulation for this case is also given for both the platoon and heat bath.

3.2. Mathematical Formulation

One notices that there is a very close analogy between traffic platoons and heat baths. A well-controlled platoon acts like an infinite reservoir whose temperature is fixed. Such a platoon may act as a reference for other relatively poorly controlled platoons. In this respect, one instantly notes that the dof of a platoon is not directly proportional to its actual number of vehicles. The interactions and information exchange determine the effective (or virtual) number of particles. This is to say that two finite but different number of vehicle platoons, when well-controlled, become infinite number of degree of freedom heat baths. The level of information exchange, or interactions, now indicates this so-called virtual number of vehicles, which is quite different from the actual number of vehicles.

For example, let us propose this virtual number of vehicles, Eq. (3), N^* as

$$N^* = f(T_r)\frac{1}{Var(T)} \tag{3}$$

where T is temperature, and $Var(T)$ is the temperature variance, for $f(T_r)$ please refer to Theorem 4.2

But since [27, 28]

$$\frac{Var(T)}{\langle T\rangle^2} = q - 1 = \frac{1}{C_v} \tag{4}$$

where q is the Tsallis entropic index, C_v is the heat capacity under constant volume. Please note that this equation is applicable for both system and the reservoir. This expression will be revised for $m \gg n$ in section 4.

After rearranging Eq. (4),

$$N^* = \frac{C_v}{\langle T\rangle^2} = \frac{1}{(q-1)\langle T\rangle^2} \tag{5}$$

Please note that this virtual dof tends to infinite reservoir dof as q tends to 1.

A platoon of vehicles, coupled to a heat bath (another platoon, for example), will have an average temperature based on their dof, Eq. (6), [24]

$$\langle T_p\rangle = T_R\frac{n(m-1)}{(n-1)(m+n)} \tag{6}$$

where *m* and *n* are virtual number of vehicles of reservoir and system, respectively. T_R is the (canonical) temperature of an infinite reservoir. $\langle T_p\rangle$ is the average temperature for the platoon (system).

$$\langle T_r\rangle = T_R\frac{m}{(m+n)} \tag{7}$$

$\langle T_r\rangle$ is the average temperature for the finite reservoir [24]. with variance

$$Var(T_r) \cong \langle T\rangle_r^2\frac{n}{m^2} \quad \text{for } m \gg n \tag{8}$$

Since for $m \gg n$, Eq. (6) is basically rearranged as

$$\langle T_p\rangle \cong T_R\frac{n}{(n-1)}.\frac{m-1}{m} \tag{9}$$

In this paper, the authors propose the following expression Eq. (10) for the platoon and infinite heat bath temperatures.

$$\langle T_p\rangle \cong T_R\frac{q_p}{q_r} \cong T_R q_p \tag{10}$$

where q_p and q_r are the Tsallis entropic indices for platoon and reservoir, respectively. And

$$T_R \cong \langle T_p\rangle\frac{q_r}{q_p} \tag{11}$$

which replaces the expression given by [35].

$$\left|\langle T_p\rangle - \langle T_r\rangle\right| = \left|T_R\frac{n(m-1)}{(n-1)(m+n)} - T_R\frac{m}{m+n}\right| \tag{12}$$

$$\Delta T = \left|\langle T_p\rangle - \langle T_r\rangle\right| = T_R\frac{m-n}{(n-1)(m+n)} \tag{13}$$

$$\Delta T \cong T_R\frac{m}{m}\frac{1}{n-1} = T_R\frac{1}{n-1} \quad for \quad m \gg n \tag{14}$$

i.e.

$$\Delta T = \frac{T_R}{n}q_p \tag{15}$$

$$n = \frac{T_R}{\Delta T}q_p = \frac{T_R}{\Delta T}(\frac{1}{n-1} + 1) \tag{16}$$

$$n = \frac{T_R}{\Delta T}(\frac{Var(T_p)}{\langle T_p\rangle^2} + 1) \tag{17}$$

Or

$$\Delta T = (q_p - 1)T_R \tag{18}$$

4. Temperature Fluctuations in Platoons of Vehicles and Results

The resources in traffic are quite limited. The future tendency may have a broader cooperation of vehicles so that well-organized groups of vehicles are controlled and moved aiming to avoid traffic congestion and hoping to improve the overall efficiency [8]. This well-coordinated movement of vehicles is meant to have a better control of the highway traffic, directed to such amenities as fluent traffic, fuel economy, automated control of vehicles, safety, desirable car following behavior etc. An organized group of vehicles is called a platoon formation where the vehicles are formed in a linear, or planar array [8]. As displayed in [8], one lane and multilane platoon formations are represented in Figure 1. and Figure 2, respectively.

Definition 4.1. Let the platoon formation is characterized by the states which are clearance (*C*), sideway (*S*), and the speed (*V*), so that $Q(C,S,V) \in R^{(m-1)\times k}$ is the combined state vector, please consider Figure 1 and Figure 2, where an m-by-n platoon as a many-body system has $(m-1)\times(n-1)$ planar dof and k designates the total number of vehicles, as previously presented in [8]. The general linear platoon formation $P(C,V) \in R^{(m-1)\times(n-1)}$ may be described as a matrix.

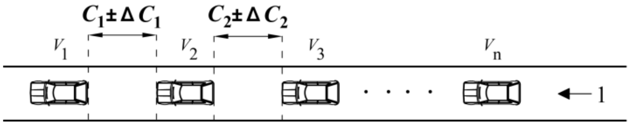

Figure 1. Representation of one lane vehicular platoon formation

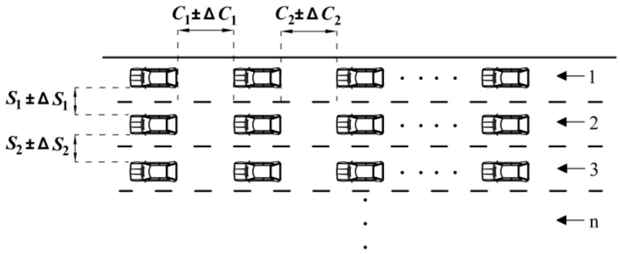

Figure 2. Representation of multilane vehicular platoon formation

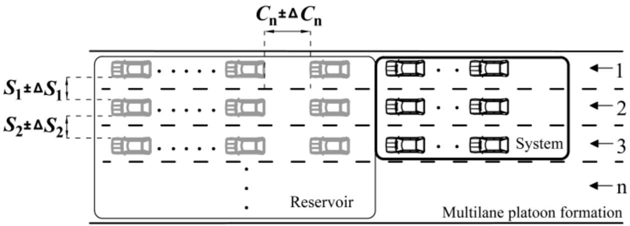

Figure 3. Representation of the canonical ensemble in terms of platoon of vehicles

Definition 4.2. Let the speed of the platoon formation be identified as temperature, and that temperature is that of a heat bath in a canonical ensemble context.

Definition 4.3. A system of platoon formation is weakly coupled to another platoon of vehicles of certain finite size together which form a canonical ensemble (Figure 3).

Definition 4.4. Let the ideal platoon formation be approached in the limit. Then the general planar platoon formation, $\mathrm{P}(C,S) \in \mathrm{R}^{(m-1)\times(n-1)}$ where an m-by-n platoon body would act as a single body, resulting in zero dof of motion solid object in plane [8].

Definition 4.5. The temperature of the platoon of vehicles of the system is defined by the larger platoon of vehicles called the heat bath whose virtual dof (number of particles) is considerably larger than the system, if not infinite.

Lemma 4.1. It should be noted that if the condition of an infinite heat bath capacity is violated, the resulting canonical distribution can no longer be of the exponential type and therefore should not follow the traditional BG thermostatistics [12, 29]. By virtue of the Kosun–Ozdemir platoon interval [8], Tsallis q index remains limited to $3>q>1$ for platoon-based vehicle or any other agent formations.

Theorem 4.1. Interactions and information exchange determine the effective (or virtual) number of particles i.e. dof. Once the platoon is perfectly controlled with no fluctuations, the influence of the physical number of particles on the temperature fluctuations would be irrelevant.

Theorem 4.2. Let each platoon be represented by a virtual number of particles (dof, or vehicles) which is separate from the actual number of vehicles. This virtual number of vehicles is a function of temperature fluctuations so that

$$N^* = f(T_r)\frac{1}{\langle (T-T_r)^2\rangle} \neq N_a \quad \text{for} \quad N^* \gg N_a \tag{19}$$

N^*= virtual number of vehicles of the given platoon, and N_a actual number of vehicles of the given platoon.

Proof Write the fluctuations for the system

$$\frac{<(\Delta T)^2>}{T_r^2} = \frac{k}{C_v} \tag{20}$$

C_v is the specific heat at T_r. Define C_v as NC_v^0, where C_v^0 is the specific heat per particle, so that the new equation becomes

$$\frac{<(\Delta T)^2>}{T_r^2} = \frac{k}{NC_v^0} \tag{21}$$

Extracting N gives

$$N_r^* = \frac{kT_r^2}{<(\Delta T)^2> C_v^0} \tag{22}$$

Or

$$N_r^* = f(T_r)\frac{1}{<(\Delta T)^2>} \tag{23}$$

where $f(T_r) = \frac{kT_r^2}{C_v^0}$

Definition 4.6. Consider a small system weakly coupled to an infinite degree of freedom heat bath. The small system exchanges heat with the heat bath, causing energy fluctuations. Since [30, 36]

$$\frac{\partial T}{\partial E} = q - 1 \tag{24}$$

for an infinite heat bath, reservoir temperature stays constant, whereas temperature fluctuates in the small system.

Lemma 4.2. A given small system experiences the minimum temperature fluctuations when coupled to an infinite degree of freedom heat bath i.e. small system temperature fluctuates even under infinite dof heat bath. Finite reservoirs would cause larger fluctuations on coupled systems.

$$inf\{<(\Delta T)^2> | N_r \to \infty\} = <(\Delta T)^2> = \frac{kT_r^2}{C_v} \tag{25}$$

where T_r , N_r are the heat bath temperature, and number of particles, respectively.

Lemma 4.3. By virtue of Definition 4.6, any small system coupled to a heat bath must have a Tsallis q value, other than 1.

Theorem 4.3. As an ideal platoon formation is approached, fluctuations (errors) in speeds (also in clearances) tend to zero. Then successive speed error readings are expected to locally relax to equilibrium at longer intervals. This state is the limiting case for any multi-component system, where all the dof is lost at $q = 1$, and for all the real platoon formations, $q > 1$, as explicitly stated in [8] and restated here.

Theorem 4.4. If the finite reservoir is sufficiently larger than the platoon coupled to it, then the reservoir acts like an infinite one.

Proof. In the light of Eq. (8) and Eq. (4), the following expression appears

$$\frac{Var(T_r)}{\langle T_r\rangle^2} \cong \frac{n}{m^2} = q - 1 \quad (26)$$

which tends to zero for $m \gg n$.

Corollary 4.1. By the same token, if the finite reservoir is sufficiently larger than the platoon coupled to it, the virtual number of dof of the finite reservoir tends to infinity. Also,

$$N_r^* \sim m^2/nT_R^2 \quad (27)$$

Lemma 4.4. If the finite reservoir is sufficiently larger than the platoon ($m \gg n$) coupled to it, Eq. (2) is revised for both platoon and the heat bath as follows

$$\frac{Var(T_r)}{\langle T_r\rangle^2} \; tends\ to \; \frac{Var(T_R)}{(T_R)^2} = \frac{k}{C_R} for\ m \gg n \quad (28)$$

$$\frac{Var(T_p)}{\langle T_p\rangle^2} \; tends\ to \; \frac{Var(T_p)}{(T_R)^2} = \frac{k}{C_p} for\ m \gg n \quad (29)$$

where C_p and C_R denote the platoon and canonical reservoir heat capacities, respectively.

Please note that as T_r tends to T_R in Eq. (28), the platoon temperature T_p remains unchanged as m tends to infinity in Eq. (29).

In summary, all of the available temperature fluctuation models in the literature are intended to be scoured. The authors were aware that there is no unique universal temperature fluctuation model. Having seen that there is no consensus on a model over temperature fluctuations, the authors tried to obtain a traffic platoon model of their own. In the model, within a canonical ensemble context, discussions on a finite system with an infinite reservoir and a finite system with a much larger but finite reservoir are provided.

In general, there are two schools of thought that could be compared. On one hand, Kittel’s view states that temperature fluctuations for finite systems in canonical ensembles do not occur, e.g. [1].

On the other hand, for example McFee states that temperature fluctuations for finite systems exist and well-posed, e.g. [3]. The other examples e.g. [12, 24, 30, 36] are also relevant for canonical ensemble with the finite heat bath approach, in line with the latter school of thought. These studies and our paper share the common assumptions since even if the system is coupled to an infinite degree of freedom reservoir, the system temperature fluctuates.

A more suitable example for our argument of a finite heat bath is first expressed by [11] where Tsallis generalized canonical distribution describes systems in contact with a finite heat bath which prevails in practice. However, the model for temperature fluctuations in this respect has not been implemented in vehicular traffic flow arena, and our study endeavored to do this.

5. Conclusions

The ramifications of temperature fluctuations concerning platoons of vehicles have been discussed. The prevalent views of different of schools of thought on temperature fluctuations were reviewed, and their connection to platoons of vehicles was elaborated. It is seen that a platoon of vehicles is in fact a system of finite vehicles in contact with a larger reservoir, which sets the average speed (temperature) for this small system, where the temperatures of both system and reservoir fluctuate. This small system and the reservoir have been assumed to form the canonical ensemble.

A so-called virtual number of vehicles is defined in terms of the 2nd moment of temperature fluctuations. A series of lemmas and definitions were given, a series of theorems was proposed. It is stated that infinite reservoirs set the temperature for the platoons. One conclusion out of this is that as the platoons are controlled better and better, their speeds tend to the specified constants so that these platoons themselves start acting like infinite heat baths whose temperatures do not fluctuate. The authors hence claim that a well-controlled or well-organized platoon of vehicles is itself a heat bath which is able to keep its temperature constant.

Furthermore, two distinct q indices for the platoon and heat bath are specified. The sizes of both the system and the reservoir are decisive in the nature of the canonical ensemble. For example, if the finite reservoir is much larger than the platoon coupled to it, then the reservoir may safely be assumed to be an infinite one. In this respect, the temperature of the finite reservoir tends to the infinite reservoir temperature.

References

[1] C. Kittel, On the nonexistence of temperature fluctuations in small systems, American Journal of Physics 41 (1973) 1211–1212.

[2] C. Kittel, Temperature fluctuation: an oxymoron, Physics Today 41 (1988) 93.

[3] R. McFee, On fluctuations of temperature in small systems, American Journal of Physics 41 (1973) 230–234.

[4] R. McFee, Comments on Professor C. Kittel's Letter on the nonexistence of temperature fluctuations in small systems, American Journal of Physics 41 (1973) 1212.

[5] Ç. Koşun, H.M. Çelik, S. Özdemir, An analysis of vehicular traffic flow using Langevin equation, Promet- Traffic & Transportation 27 (2015) 317–324.

[6] Ç. Koşun, The use of nonextensive framework in connection with traffic flow [PhD dissertation], Department of City and Regional Planning, Izmir Institute of Technology Graduate School of Engineering and Sciences, Izmir, 2015.

[7] C. Kosun, S. Ozdemir, A superstatistical model of vehicular traffic flow, Physica A: Statistical Mechanics and its Applications 444 (2016) 466–475.

[8] C. Kosun, S. Ozdemir, Determining the complexity of multi-component conformal systems: A platoon-based approach, Physica A: Statistical Mechanics and its Applications 471 (2017) 688–695.

[9] M. Krbálek, D. Helbing, Determination of interaction potentials in freeway traffic from steady-state statistics, Physica A: Statistical Mechanics and its Applications 333 (2004) 370–378.

[10] R.G. Kulkarni, R.R. Stough, K.E. Haynes, Spin glass and the interactions of congestion and emissions: an exploratory step, Transportation Research Part C: Emerging Technologies 4 (1996) 407–424.

[11] A.R. Plastino, A. Plastino, From Gibbs microcanonical ensemble to Tsallis generalized canonical distribution, Physics Letters A 193 (1994) 140–143.

[12] F.Q. Potiguar, U.M.S. Costa, Numerical calculation of the energy-relative fluctuation for a system in contact with a finite

heat bath, Physica A: Statistical Mechanics and its Applications 342 (2004) 145–150.
[13] T.S. Biró, G.G. Barnaföldi, P. Ván, New entropy formula with fluctuating reservoir, Physica A: Statistical Mechanics and its Applications 417 (2015) 215–220.
[14] D. Su, S. Ahn, Autonomous platoon formation for VANET-enabled vehicles. In: International Conference: Information and Communication Technology Convergence (ICTC), Jeju, South Korea, October 19–21 (2016).
[15] Q. Deng, X. Ma, A fast algorithm for planning optimal platoon speeds on highway, IFAC Proceedings Volumes 47 (2014) 8073–8078.
[16] S. Santini, A. Salvi, A.S. Valente, A. Pescapè, M. Segata, R.L. Cigno, A consensus-based approach for platooning with inter-vehicular communications. In: IEEE Conference: Computer Communications (INFOCOM), Kowloon, Hong Kong, April 26–May 1 (2015).
[17] J.W. Gibbs, Elementary Principles in Statistical Physics, Yale University Press, 1902.
[18] L.D. Landau, E.M. Lifshitz. Course of Theoretical Physics Vol. 5, Statistical Physics, Part I, 3rd ed., Pergamon Press, 1980.
[19] T.C.P. Chui, D.R. Swanson, M.J. Adriaans, J.A. Nissen, J.A. Lipa, Temperature fluctuations in the canonical ensemble, Physical Review Letters 69 (1992) 3005.
[20] B.B. Mandelbrot, Temperature fluctuation: A well-defined and unavoidable notion, Physics Today 42 (1989) 71–73.
[21] M. Falcioni, D. Villamaina, A. Vulpiani, A. Puglisi, A. Sarracino, Estimate of temperature and its uncertainty in small systems, American Journal of Physics 79 (2011) 777–785.
[22] J. Hickman, Y. Mishin, Temperature fluctuations in canonical systems: insights from molecular dynamics simulations, Physical Review B 94 (2016) 184311.
[23] J. Lindhard, Complementarity Between Energy and Temperature, In The Lesson of Quantum Theory (Eds. J. de Boer, E. Dal, O. Ulfbeck), North-Holland, Amsterdam, 1986.
[24] H.B. Prosper, Temperature fluctuations in a heat bath, American Journal of Physics 61 (1993) 54-58.
[25] L. Stodolsky, Temperature fluctuations in multiparticle production, Physical Review Letters 75 (1995) 1044.
[26] J.L. van Hemmen, A. Longtin, Temperature fluctuations for a system in contact with a heat bath, Journal of Statistical Physics 153 (2013) 1132-1142.
[27] G. Wilk, Z. Włodarczyk, Quasi-power law ensembles, arXiv preprint arXiv:1501.01936 (2015).
[28] G. Wilk, Z. Włodarczyk, Multiplicity fluctuations due to the temperature fluctuations in high-energy nuclear collisions, Physical Review C 79 (2009) 054903.
[29] A.B. Adib, A.A. Moreira, J.S. Andrade Jr, M.P. Almeida, Tsallis thermostatistics for finite systems: a Hamiltonian approach, Physica A: Statistical Mechanics and its Applications 322 (2003) 276–284.
[30] M.P. Almeida, Generalized entropies from first principles, Physica A: Statistical Mechanics and its Applications 300 (2001) 424–432.
[31] T. Wada, Model-free derivations of the Tsallis factor: constant heat capacity derivation, Physics Letters A 318 (2003) 491–498.
[32] S. Abe, A.K. Rajagopal, Macroscopic thermodynamics of equilibrium characterized by power law canonical distributions, EPL (Europhysics Letters) 55 (2001) 6.
[33] G. Wilk, Z. Włodarczyk, Equivalence of volume and temperature fluctuations in power-law ensembles, Journal of Physics G: Nuclear and Particle Physics 38 (2011) 065101.
[34] G. Wilk, Z. Włodarczyk, Interpretation of the nonextensivity parameter q in some applications of Tsallis statistics and Lévy distributions, Physical Review Letters 84 (2000) 2770.
[35] J.D. Ramshaw, Entropy ambiguity in a system in equilibrium with a finite heat bath, Physics Letters A 198 (1995) 122–125.
[36] F.Q. Potiguar, U.M.S Costa, Fluctuation of energy in the generalized thermostatistics, Physica A: Statistical Mechanics and its Applications 321 (2003) 482–492.